Solutions

1. **Tags: Algebra**

D,

We need to determine the number of different solutions, including both real and complex numbers, to the equation:

$$x^{2025} = 1$$

The equation $x^{2025} = 1$ means we are looking for all complex solutions where x raised to the power of 2025 equals 1. These solutions are the 025th roots of unity.

The general form of the nth roots of unity is given by:

$$x_k = e^{2\pi ik/n}, \quad k = 0, 1, 2, \ldots, n-1.$$

For our specific case where $n = 2025$, we get:

$$x_k = e^{2\pi ik/2025}, \quad k = 0, 1, 2, \ldots, 2024.$$

Each value of k from 0 to 2024 provides a distinct solution. Since there are 2025 values of k, there are exactly:

$$\mathbf{2025}$$

distinct solutions to the equation $x^{2025} = 1$, including both real and complex solutions.

2. **Tags: Geometry**

D,

Apply Heron's formula

$$s = \frac{a+b+c}{2} = 11$$
$$A = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{440}$$

3. **Tags: Counting**

E,

The total number of ways to choose 3 members is $c(20, 3)$. But if Alice is an officer, Bob cannot be chosen, we have to choose from the other 18 members. Therefore altogether we have

$$c(20, 3) - c(18, 1)$$

Once we have chosen the 3 members, they can act as different roles, therefore

$$N = (c(20, 3) - c(18, 1)) * p(3, 3) = (\frac{20!}{3!17!} - 18) * 3! = 6732$$

4. **Tags: Calculus**

E,

To determine how fast the spot of light is moving along the ground, we define the following variables:

Let θ be the angle between the lamp's beam and the line directly downward (i.e., the vertical post).
Let x be the horizontal distance from the base of the post to the spot of light on the ground.
The lamp is mounted on a 3-meter-high post.

Using right triangle trigonometry:

$$\tan\theta = \frac{x}{3}$$

Solving for x:

$$x = 3\tan\theta$$

Differentiating both sides with respect to time t:

$$\frac{dx}{dt} = 3\sec^2\theta\frac{d\theta}{dt}$$

We are given that the rate of change of the angle is:

$$\frac{d\theta}{dt} = 0.1 \text{ radians per second}$$

We are given that the light initially shines 10 meters away from the base, so:

$$10 = 3\tan\theta$$

Solving for $\tan\theta$:

$$\tan\theta = \frac{10}{3}$$

Using the identity:

$$\sec^2\theta = 1 + \tan^2\theta$$
$$\sec^2\theta = 1 + \left(\frac{10}{3}\right)^2$$
$$\sec^2\theta = 1 + \frac{100}{9} = \frac{109}{9}$$
$$\frac{dx}{dt} = 3 \times \frac{109}{9} \times 0.1$$
$$\frac{dx}{dt} = \frac{327}{90}$$
$$\frac{dx}{dt} \approx 3.63 \text{ meters per second}$$

The spot of light is moving along the ground at approximately 3.63 meters per second when it is 10 meters away from the base of the post.

In summary

$$v = \frac{dx}{dt} = \frac{dh\tan\theta}{dt} = h\sec^2\theta\frac{d\theta}{dt} = \frac{h}{\cos^2\theta}\omega = \frac{3}{3^2/(3^2+10^2)}\cdot 0.1 = 3.63\,\text{m/s}$$

5. **Tags: Markov Chain**

E,

To find the distribution of cars after one transition step, we multiply the initial distribution vector X_0 by the transition matrix P:

$$X_1 = PX_0$$

where:

$$P = \begin{bmatrix} 0 & 0.6 & 0.4 \\ 0.5 & 0 & 0.5 \\ 0.3 & 0.7 & 0 \end{bmatrix}$$

and

$$X_0 = \begin{bmatrix} 300 \\ 400 \\ 200 \end{bmatrix}$$

The new number of cars in each city is given by:

$$X_1 = \begin{bmatrix} 0 & 0.6 & 0.4 \\ 0.5 & 0 & 0.5 \\ 0.3 & 0.7 & 0 \end{bmatrix} \begin{bmatrix} 300 \\ 400 \\ 200 \end{bmatrix}$$

Computing each entry:

1. Cars in City A after one step:

$$0(300) + 0.6(400) + 0.4(200) = 0 + 240 + 80 = 320$$

2. Cars in City B after one step:

$$0.5(300) + 0(400) + 0.5(200) = 150 + 0 + 100 = 250$$

3. Cars in City C after one step:

$$0.3(300) + 0.7(400) + 0(200) = 90 + 280 + 0 = 370$$

Thus, the new distribution of cars is:

$$X_1 = \begin{bmatrix} 320 \\ 250 \\ 370 \end{bmatrix}$$

After one transition step, the number of cars in each city is:
320 cars in City A
250 cars in City B
370 cars in City C

6. **Tags: Dynamics, Rotation, Kinematics**

D,

$$a = \frac{g \sin\theta}{1+\beta}$$
$$\beta_4 < \beta_1 = \beta_2 < \beta_3$$

Therefore

$$a_4 > a_1 = a_2 > a_3$$
$$t_4 < t_1 = t_2 < t_3$$

7. **Tags: Thermodynamics**

C,

The coefficient of performance (COP) for a refrigerator is given by the formula:

$$COP = \frac{T_L}{T_H - T_L}$$

where: T_L is the lower temperature (inside the refrigerator),
T_H is the higher temperature (garage temperature),
Temperatures must be converted to Kelvin.

$$T_L = -5 + 273 = 268K$$
$$T_H = -10 + 273 = 263K$$

$$COP = \frac{268}{263 - 268}$$
$$COP = \frac{268}{-5}$$
$$COP = -53.6$$

Since the result is egative this indicates that the refrigerator is actually working gainst natural heat flow which means it's acting more like a heater rather than a cooling device in this case. Normally, a refrigerator should reject heat to a hotter environment, but here, the garage is colder than the refrigerator itself, making it inefficient.

8. **Tags: Electrostatics**

A,

The electric potential V at a point due to a charge distribution is given by:

$$V = \int \frac{dq}{4\pi\epsilon_0 r}$$

where dq represents charge elements and r is the distance from the charge element to the point where the potential is being calculated.

For a uniformly charged sphere:

1. Outside the sphere ($r \geq R$): The sphere behaves like a point charge with total charge Q, so the potential at a distance r from the center is:

$$V_{\text{outside}}(r) = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$$

2. On the surface ($r = R$):

$$V(R) = \frac{1}{4\pi\epsilon_0}\frac{Q}{R}$$

3. Inside the sphere ($r < R$): The potential at any point inside a uniformly charged sphere is given by:

$$V_{\text{inside}}(r) = \frac{1}{4\pi\epsilon_0}\frac{Q}{R}\left(\frac{3}{2} - \frac{r^2}{2R^2}\right)$$

Substituting $r = 0$ into the equation for $V_{\text{inside}}(r)$:

$$V(0) = \frac{1}{4\pi\epsilon_0}\frac{Q}{R} \times \frac{3}{2}$$

$$V(0) = \frac{3Q}{8\pi\epsilon_0 R}$$

The electric potential at the center of the sphere is:

$$V(0) = \frac{3Q}{8\pi\epsilon_0 R}$$

9. **Tags: Optics**

D,

The magnification (m) of a mirror is given by:

$$m = \frac{\text{image height}}{\text{object height}} = \frac{-d_i}{d_o}$$

where: d_o is the object distance (distance of Bob's nose from the mirror),
d_i is the image distance (distance of the nose's image from the mirror),
m is given as $\frac{1}{2}$ (half the size),
The mirror equation is:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

The image is upright, which means the mirror is a convex mirror (because concave mirrors produce upright images only when the object is inside the focal point, which would result in magnification greater than 1). The focal length is negative for convex mirrors:

$$f = -20 \text{ cm}$$

The magnification formula:

$$m = -\frac{d_i}{d_o}$$

Given that $m = \frac{1}{2}$:

$$\frac{1}{2} = -\frac{d_i}{d_o}$$
$$d_i = -\frac{d_o}{2}$$

(The negative sign confirms that the image is virtual, which is expected for a convex mirror.)

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

Substituting $f = -20$ cm and $d_i = -\frac{d_o}{2}$:

$$\frac{1}{-20} = \frac{1}{d_o} + \frac{1}{-\frac{d_o}{2}}$$
$$\frac{1}{-20} = \frac{1}{d_o} - \frac{2}{d_o}$$
$$\frac{1}{-20} = \frac{-1}{d_o}$$
$$d_o = 20 \text{ cm}$$

Bob's nose is 20 cm away from the mirror.

10. **Tags: Special Relativity**

A,

This is a time dilation problem from special relativity. The time dilation formula is:

$$\Delta t' = \frac{\Delta t}{\gamma}$$

where: $\Delta t'$ is the proper time experienced by Bob (time elapsed on his spaceship),
Δt is the time elapsed in Adam's frame (time on Earth),
γ is the Lorentz factor, given by:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Given $v = \frac{3}{5}c$, we calculate:

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{3}{5}\right)^2}}$$
$$\gamma = \frac{1}{\sqrt{1 - \frac{9}{25}}} = \frac{1}{\sqrt{\frac{16}{25}}} = \frac{1}{\frac{4}{5}} = \frac{5}{4}$$

The time elapsed in Adam's frame is:

$$\Delta t = 40 - 20 = 20 \text{ years}$$

Applying time dilation:

$$\Delta t' = \frac{20}{\gamma} = \frac{20}{\frac{5}{4}} = 20 \times \frac{4}{5} = 16 \text{ years}$$

Thus, Bob experiences 16 years while Adam experiences 20 years.

Bob starts at 20 years old, so his age after experiencing 16 years is:

$$20 + 16 = 36 \text{ years}$$

According to Adam, when Adam is 40 years old, Bob is 36 years old.

11. **Tags: Stoichiometry**

B,

To determine the volume of oxygen gas consumed when a 2-gram sample of iron reacts with oxygen to form a mixture of 30% iron(II) oxide (FeO) and 70% iron(III) oxide (Fe_2O_3) by mass, we follow these steps:

Let M be the total mass of the products.
The mass of FeO is $0.3M$ and the mass of Fe_2O_3 is $0.7M$.
The mass of iron in FeO is $0.3M \times \frac{55.85}{71.85}$.
The mass of iron in Fe_2O_3 is $0.7M \times \frac{2\times55.85}{159.7}$.
The total mass of iron from both oxides must equal 2 grams:

$$0.3M \times \frac{55.85}{71.85} + 0.7M \times \frac{111.7}{159.7} = 2$$

Solving this equation gives $M \approx 2.766$ grams.

The mass of oxygen in the products is the total mass of the products minus the mass of iron:

$$2.766\,\text{g} - 2\,\text{g} = 0.766\,\text{g}$$

The molar mass of O_2 is 32 g/mol:

$$\text{Moles of } O_2 = \frac{0.766\,\text{g}}{32\,\text{g/mol}} \approx 0.02394\,\text{mol}$$

At STP, 1 mole of gas occupies 22.4 liters:

$$\text{Volume} = 0.02394\,\text{mol} \times 22.4\,\text{L/mol} \approx 0.536\,\text{L}$$

12. **Tags: Chemical Kinetics**

B,

The general rate law for a reaction involving reactant A is:

$$\text{Rate} = k[A]^n$$

where:
k is the rate constant,
$[A]$ is the concentration of reactant A,
n is the order of the reaction with respect to A.

For different reaction orders, the integrated rate laws have specific forms:

1. Zero-order reaction ($n = 0$):

$$[A] = [A]_0 - kt$$

(A plot of [A] vs. t is linear.)

2. First-order reaction ($n = 1$):

$$\ln[A] = \ln[A]_0 - kt$$

(A plot of $\ln[A]$ vs. t is linear.)

3. Second-order reaction ($n = 2$):

$$\frac{1}{[A]} = \frac{1}{[A]_0} + kt$$

(A plot of $\frac{1}{[A]}$ vs. t is linear.)

The problem states that the logarithm of the concentration of A is linearly dependent on time. This matches the first-order integrated rate law:

$$\ln[A] = \ln[A]_0 - kt$$

Since this equation represents a straight-line graph with slope $-k$, the reaction must be first-order with respect to A.

The reaction is first-order ($n = 1$) with respect to reactant A.

13. **Tags: Chemical Thermodynamics**

A,

At the midpoint of a titration involving a weak base and a strong acid, half of the base has been neutralized. This means the concentrations of the base (B) and its conjugate acid (BH^+) are equal. At this point, the pH of the solution is determined by the K_b of the weak base and the pKa of its conjugate acid.

The relevant relationship at the midpoint for the weak base is given by the equation for the buffer:

$$\text{pH} = \text{pKa} + \log\left(\frac{[\text{B}]}{[\text{BH}^+]}\right)$$

Since the concentrations of B and BH^+ are equal at the midpoint:

$$\text{pH} = \text{pKa}$$

Thus, pKa of the conjugate acid is equal to the pH at the midpoint of the titration, which is 10.

Now, we can find the base dissociation constant (K_b) using the relationship between pKa and pKb for conjugate acid-base pairs:

$$\text{pKa} + \text{pKb} = 14$$

Since the pKa = 10:

$$10 + \text{pKb} = 14$$

$$\text{pKb} = 4$$

Now, we can find the K_b from:

$$K_b = 10^{-\text{pKb}} = 10^{-4}$$

Thus, the base dissociation constant is:

$$K_b = 1 \times 10^{-4}$$

The base dissociation constant (K_b) of the weak base is 1×10^{-4}.

14. **Tags: Electrochemistry**

    B,

    To calculate the cell potential under non-standard conditions, we use the Nernst equation:

$$E = E^{\circ} - \frac{0.0592}{n} \ln Q$$

Where:
E is the cell potential at non-standard conditions,
E° is the standard cell potential (1.2 V),
n is the number of moles of electrons transferred in the reaction (here, $n = 2$),
Q is the reaction quotient (given as 7.389),
The factor 0.0592 is used at 298 K (standard temperature).

We know:
$E^{\circ} = 1.2\,\text{V}$,
$n = 2$,
$Q = 7.389$.

Substitute into the Nernst equation:

$$E = 1.2 - \frac{0.0592}{2} \ln(7.389)$$
$$\ln(7.389) = 2$$

Now substitute this into the equation:

$$E = 1.2 - \frac{0.0592}{2} \times 2$$
$$E = 1.2 - 0.0592$$
$$E = 1.1408\,\text{V}$$

The cell potential when the reaction quotient Q is 7.389 is approximately 1.14 V.

15. **Tags: Molecular Geometry**

    C,

    It's T-shaped when the VESPR numbers are 532.

16. **Tags: Stars**

    E,

    The parallax angle tells us the distance

$$d = \frac{1AU}{p} = 500\,\text{pc}$$

17. **Tags: Astronomy**

    C,

    Maximum altitude occurs on the day of summer solstice,

$$a = 90 - \phi + \epsilon = 90 - 40 + 23.5 = 73.5^\circ$$

18. **Tags: Astronomy**

    B,

19. **Tags: Stars**

    E,

    Distance modulus formula

$$m - M = 5\log d - 5$$
$$m' - M = 5\log d' - 5$$
$$m' - m = 5\log d' - 5\log d = 5\log 10 = 5$$
$$m' = m + 5 = 6.5$$

20. **Tags: Orbital Mechanics**

    A,

$$E = -\frac{GMm}{2R}$$
$$E + \delta E = -\frac{GMm}{2R + \delta R} = -\frac{GMm}{2R}(1 - \delta R/R)$$
$$\delta R = \frac{2\delta E R^2}{GMm} \approx 0.1\,\mathrm{m}$$
$$T^2/R^3 = (T + \delta T)^2/(R + \delta R)^3 \approx T^2/R^3(1 + 2\delta T/T)(1 - 3\delta R/R)$$
$$2\delta T/T - 3\delta R/R = 0$$
$$\delta T = \frac{3\delta R T}{2R} = \frac{3\delta E R}{GMm}T$$
$$\delta E = \frac{1}{2}m(v^2 + \delta v^2) - \frac{1}{2}mv^2 = \frac{1}{2}m\delta v^2$$
$$\delta T = \frac{3\frac{1}{2}m\delta v^2 R}{GMm}T = \frac{3\delta v^2 R}{2GM}T = 1.33 \times 10^{-4}\,\mathrm{s}$$
$$v = \frac{2\pi R}{T} = 7623\,\mathrm{m/s}$$
$$v\delta T = 1.16\,\mathrm{m}$$

21. **Tags: C**

    D,

    This generates an out of the bound error, ptr[4] is fine.

22. **Tags: Python**

    E,

    It's missing the colon at the end of the line.

	Space Complexity	Time Complexity
Merge sort	O(n)	O($n \ln n$))
Cycle sort	O(1)	O(n^2)

23. **Tags: Algorithm**

B,

(a) Option A: Incorrect. Time complexity (Big O notation) describes asymptotic behavior but ignores constants and practical differences. For example, two O($n \log n$) sorting algorithms (e.g., Merge Sort vs. Quick Sort) can perform differently due to implementation details.

(b) Option B: Correct. In C, for loops are syntactic sugar for while loops. A while loop can be rewritten as a for loop by omitting the initialization and increment expressions

(c) Option C: Incorrect. Floating-point numbers are approximations due to finite precision (e.g., rounding errors in binary representation).

(d) Option D: Incorrect. Adding threads introduces overhead (e.g., synchronization, context switching). Beyond a certain point, more threads degrade performance (Amdahl's Law).

(e) Option E: Incorrect. Hashing (even cryptographic hashing) does not fully secure data. Weak hashing algorithms, collisions, and lack of encryption make hashed data vulnerable.

24. **Tags: Algorithm**

C,

In the given methods only merge sort and cylce sort sort the integers. Using Cycle Sort (or a similar in-place swapping approach) allows you to sort the array in O(n^2) time and O(1) space, which meets the problem's constraints.

25. **Tags: Algorithm**

C,

(a) Option C describes a recursive Depth-First Search (DFS) approach where each node checks if it is the LCA by determining if both target nodes are found in its left and right subtrees. This method efficiently traverses the tree once, resulting in a worst-case time complexity of O(n).

(b) Option A involves two DFS traversals to find paths and then compares them. While this is O(n) time, it requires additional space to store paths and involves more steps compared to the single traversal in Option C.

(c) Option B is invalid because converting a binary tree into a BST is not feasible in O(n) time and alters the tree structure, which is not permitted.

(d) Option D suggests using BFS from both nodes, which is impractical without parent pointers and does not guarantee O(n) efficiency.

Option C's recursive DFS approach directly finds the LCA in a single traversal, making it the most efficient O(n) solution.

26. 1.3125

Tags: Algebra

We are given the polynomial:

$$f(x) = x^3 - 7x^2 + 14x - 8$$

with roots a, b, c, and we need to evaluate:

$$S = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}$$

We use the identity:

$$\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{(b^2c^2 + c^2a^2 + a^2b^2)}{(a^2b^2c^2)}$$

Rewriting using Vieta's formulas:
Sum of roots: $a + b + c = 7$
Sum of product of roots taken two at a time: $ab + bc + ca = 14$
Product of roots: $abc = 8$

We use the identity:

$$a^2b^2 + b^2c^2 + c^2a^2 = (ab + bc + ca)^2 - 2abc(a + b + c)$$

Substituting known values:

$$a^2b^2 + b^2c^2 + c^2a^2 = 14^2 - 2(8)(7)$$

$$= 196 - 112 = 84$$

Since:

$$a^2b^2c^2 = (abc)^2 = 8^2 = 64$$

$$S = \frac{84}{64} = \frac{21}{16}$$

Thus, the final answer is:

$$\boxed{\frac{21}{16}} \approx 1.3125$$

27. 1.33

Tags: Dynamics

To find the acceleration of the wedge, we start by considering the forces and accelerations acting on both the block and the wedge. The key steps are as follows:

1. Define the accelerations:
Let A be the acceleration of the wedge to the left.
Let a be the acceleration of the block relative to the wedge along the incline.

2. Resolve the accelerations:
The horizontal component of the block's acceleration relative to the wedge is $a\cos(30°)$.
The vertical component of the block's acceleration relative to the wedge is $a\sin(30°)$.

3. Apply Newton's second law:
For the block in the horizontal direction: $N\sin(30^\circ) = m_{\text{block}}(-A + a\cos(30^\circ))$.
For the block in the vertical direction: $N\cos(30^\circ) - m_{\text{block}}g = -m_{\text{block}}a\sin(30^\circ)$.
For the wedge in the horizontal direction: $N\sin(30^\circ) = m_{\text{wedge}}A$.

4. Solve the system of equations:
From the horizontal equations: $m_{\text{wedge}}A = m_{\text{block}}(-A + a\cos(30^\circ))$.
Substitute N from the wedge equation into the block's vertical equation.

5. Use the center of mass approach:
The horizontal center of mass must remain stationary, leading to the equation $m_{\text{block}}(-A + a\cos(30^\circ)) + m_{\text{wedge}}(-A) = 0$.

6. Combine and simplify the equations:
Solve for a in terms of A.
Substitute back to find A.

After solving the equations and substituting the given values (mass of the block 1 kg, mass of the wedge 3 kg, angle 30°, and gravitational acceleration $g = 10\,\text{m/s}^2$), we find the acceleration of the wedge:

$$A = \frac{g\sin(30^\circ)\cos(30^\circ)}{m_{\text{wedge}} + m_{\text{block}}\sin^2(30^\circ)}$$

Substituting the values:

$$A = \frac{10 \cdot 0.5 \cdot \frac{\sqrt{3}}{2}}{3 + 1 \cdot (0.5)^2} = \frac{10 \cdot 0.433}{3.25} \approx 1.33\,\text{m/s}^2$$

28. -0.85

Tags: Electrochemistry

We need to find E^o of the reaction:

$$\text{Cr}^{2+} + 2\text{e}^- \longrightarrow \text{Cr(s)}$$

Remembering that: $\Delta G^o = -nFE^o$ and $F = 96\,500\,\text{C/mol}$, it is possible to find the ΔG^o values of the given reactions:

$$\Delta G^o(1) = -3 \times F \times (-0.73) = 211.(\text{kJ/mol})$$
$$\Delta G^o(2) = -1 \times F \times (-0.50) = 48(\text{ kJ/mol})$$

The ΔG^o of the reaction that we investigate is:

$$\Delta G^o(3) = \Delta G^o(1) - \Delta G^o(2) = 163(\text{ kJ/mol})$$

Using the equation: $\Delta G^o = -nFE^o$ and $n = 2$, again we find that $E^o(3) = -0.85$ V.

29. 1.24e9

Tags: Stars

To obtain the electron pressure in function of the density, we can use that $n = N_e/V = ZN/V$, where N is the number of atoms in the star. Since we can consider that just the protons and the neutrons contribute to the mass of an atom, we can say that $N = \frac{M}{Am_p}$ and so $n = \frac{Z\rho}{Am_p}$. For the given problem, $Z = 6, A = 12$ which is carbon. Besides that, since $\rho = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi R^3}$,

$$R^3 = (\frac{3M}{4\pi\rho})^{1/3}$$

Therefore, we can balance the electron and the gravitational pressure (the condition for the existence of the white dwarf) to obtain the value of the density:

$$\begin{aligned}\rho &= \frac{4G^3M^2m_e^3}{27\pi^3\hbar^6}(\frac{Am_p}{Z})^5 \\ &= \frac{4(6.67*10^{-11})^3(1.989*10^{30})^2(9.11*10^{-31})^3}{27\pi^3*(6.67*10^{-34}/(2*\pi))^6}(\frac{12*1.6726*10^{-27}}{6})^5 \\ &\approx 1.24\times 10^9\,\mathrm{kg/m^3}\end{aligned}$$

This is incredible density only countered by electron degeneracy pressure.

30. -66

Tags: Algorithm

The output using the given list:

-10 20 31 41 -38 12 82 -27 73 -66 34 -45 15 56 -53 67 48 -91 89 99